Proposition 5 (ii) is deduced from

$$\Lambda_t = \Lambda_{R_1} + \Lambda_{R_2} \quad \text{and} \quad \Lambda_0 = \Lambda_{R_1} \cap \Lambda_{R_2},$$

which are verified as for (i).

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# Univalent (Monodentate) Substitution on Convex Polyhedra. II. Listing of Cycle Indices

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To extend the usefulness of the tabulation of the numbers $N$ of positional isomers [Knop, Barker & White (1975). *Acta Cryst.* A**31**, 461–472], all the distinct cycle-index polynomials $Z$ on which the tabulation is based have been listed in a convenient form. This condensed summary facilitates identification of $Z$-isomorphisms; in turn, $N$ for univalent substitution on many polyhedra not listed previously can be evaluated simply by reference to the existing tabulation.

In part I (Knop, Barker & White, 1975) we presented the numbers $N$ of distinct (up to rotation) positional isomers obtained by univalent substitution at the vertices of convex polyhedra; only structureless substituents were considered. The tabulation is extensive, but naturally it cannot include *all* non-isomorphic polyhedra even for small numbers of vertices $V$. A user of the tables wishing to evaluate $N$ for polyhedra not listed in Table 5 of part I would not only have to determine the appropriate cycle indices $Z$, but he would have to compute the coefficients of the expanded cycle-index polynomials (*i.e.* the values of $N$) for the compositions of interest, a tedious task. However, owing to cycle-index isomorphism the number of distinct $Z$ polynomials involved in the tabulations of part I is not unduly large, and there is a good chance that the set of $N$ to be determined already appears there under a different but $Z$-isomorphic polyhedron, which makes fresh computation unnecessary.

To facilitate identification of additional $Z$-isomorphisms, over and above those *specifically* listed in part I, a table of *all* the cycle indices on which the tabulation of part I is based, has been compiled.

Considerable space is saved by introducing the following notation. An $s$-product $s_a^u s_b^v$ will be represented as $a,u{*}b,v$. Each $s$-product occurring in the cycle indices for a particular value of $V$ will be denoted by a capital letter (Table 1). The highest-order term $s_1^V$ (represented by $A$) is always present,† and so further space is saved by omitting $A$ from the letter symbol of $Z$. For example, the $Z$ of a tetrahedron **4–2** of symmetry $T_d$,

$$\tfrac{1}{24}(s_1^4 + 6s_4^1 + 8s_1^1s_3^1 + 6s_1^2s_2^1 + 3s_2^2),$$

is represented by $6B8C6D3E$. The $s$-products denoted by the letters are found in Table 1 under $V=4$. The sum of the coefficients associated with the letters, including the coefficient of $A$, which is always unity, is equal to the divisor $p(\mathbf{G})$, in this case 24.

For economy of space, the table of cycle indices (Table 2) is arranged as follows. In the first part (pp. 3–9) $Z$ polynomials occurring in only a few cases are listed in the order of increasing $V$. The second part (pp. 9–16) contains cycle indices having large numbers of terms and those involved in considerable numbers of $Z$-isomorphic representations.

† The term $A$ by itself represents $Z(4mh)$ of the corresponding polygon of $V$ vertices (*cf.* part I).

‡ Table 2 has been deposited with the British Library Lending Division as Supplementary Publication No. SUP 31246 (16 pp., 1 microfiche). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 13 White Friars, Chester CH1 1NZ, England.

Table 1. *Cycle-index terms*

| *V* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *J* |
|---|---|---|---|---|---|---|---|---|
| 3 | 3,1 | 1,1*2,1 | | | | | | |
| 4 | 4,1 | 1,1*3,1 | 1,2*2,1 | 2,2 | | | | |
| 5 | 5,1 | 1,1*4,1 | 2,1*3,1 | 1,2*3,1 | 1,1*2,2 | 1,3*2,1 | | |
| 6 | 6,1 | 1,1*5,1 | 2,1*4,1 | 1,2*4,1 | 3,2 | 1,2*2,2 | 1,4*2,1 | 2,3 |
| 7 | 7,1 | 1,1*6,1 | 2,1*5,1 | 1,2*5,1 | 1,1*3,2 | 1,1*2,3 | 1,3*2,2 | 1,5*2,1 |
| 8 | 8,1 | 1,1*7,1 | 2,1*6,1 | 1,2*6,1 | 2,1*3,2 | 4,2 | 1,2*3,2 | 1,2*2,3 |
| 9 | 9,1 | 1,1*8,1 | 2,1*7,1 | 1,2*7,1 | 3,1*6,1 | 1,1*4,2 | 3,3 | 1,1*2,4 |
| 10 | 10,1 | 1,1*9,1 | 2,1*8,1 | 1,2*8,1 | 2,1*4,2 | 5,2 | 1,2*4,2 | 1,1*3,3 |
| 11 | 11,1 | 1,1*10,1 | 2,1*9,1 | 1,2*9,1 | 2,1*3,1*6,1 | 1,1*5,2 | 2,1*3,3 | 1,2*3,3 |
| 12 | 12,1 | 1,1*11,1 | 2,1*10,1 | 1,2*10,1 | 2,1*5,2 | 6,2 | 1,2*5,2 | 4,3 |
| 13 | 13,1 | 1,1*12,1 | 2,1*11,1 | 1,2*11,1 | 1,1*6,2 | 1,1*4,3 | 1,1*3,4 | 1,1*2,6 |
| 14 | 14,1 | 1,1*13,1 | 2,1*12,1 | 1,2*12,1 | 2,1*3,2*6,1 | 2,1*6,2 | 7,2 | 1,2*6,2 |
| 15 | 15,1 | 2,1*13,1 | 5,1*10,1 | 1,2*13,1 | 3,1*6,2 | 5,3 | 3,5 | 1,1*2,7 |
| 16 | 16,1 | 8,2 | 4,4 | 1,1*3,5 | 1,2*2,7 | 1,4*2,6 | 2,8 | |
| 17 | 17,1 | 1,1*2,8 | | | | | | |
| 18 | 18,1 | 9,2 | 6,3 | 2,1*4,4 | 1,2*4,4 | 3,6 | 1,2*2,8 | 1,4*2,7 |
| 19 | 19,1 | 1,1*2,9 | | | | | | |
| 20 | 10,2 | 2,1*6,3 | 5,4 | 4,5 | 1,2*3,6 | 1,2*2,9 | 1,4*2,8 | 1,6*2,7 |
| 24 | 12,2 | 8,3 | 6,4 | 4,6 | 3,8 | 1,2*2,11 | 1,4*2,10 | 1,8*2,8 |
| 26 | 2,1*6,4 | 2,1*4,6 | 1,2*4,6 | 1,2*3,8 | 1,2*2,12 | 1,8*2,9 | 2,13 | |
| 30 | 10,3 | 6,5 | 5,6 | 3,10 | 1,2*2,14 | 1,4*2,13 | 2,15 | |
| 32 | 2,1*10,3 | 2,1*6,5 | 1,2*5,6 | 1,2*3,10 | 1,8*2,12 | 2,16 | | |
| 38 | 1,2*4,9 | 1,2*3,12 | 1,2*2,18 | 2,19 | | | | |
| 48 | 6,8 | 4,12 | 3,16 | 2,24 | | | | |
| 60 | 10,6 | 6,10 | 5,12 | 3,20 | 1,4*2,28 | 2,30 | | |
| 62 | 2,1*10,6 | 2,1*6,10 | 1,2*5,12 | 1,2*3,20 | 1,2*2,30 | 1,12*2,25 | 2,31 | |
| 92 | 1,2*5,18 | 1,2*3,30 | 2,46 | | | | | |
| 120 | 10,12 | 6,20 | 5,24 | 3,40 | 2,60 | | | |

| *V* | *K* | *L* | *M* | *N* | *P* | *Q* | *R* | *S* | *T* |
|---|---|---|---|---|---|---|---|---|---|
| 8 | 1,4*2,2 | 1,6*2,1 | 2,4 | | | | | | |
| 9 | 1,3*2,3 | 1,5*2,2 | 1,7*2,1 | | | | | | |
| 10 | 1,2*2,4 | 1,4*2,3 | 1,6*2,2 | 1,8*2,1 | 2,5 | | | | |
| 11 | 1,1*2,5 | 1,3*2,4 | 1,5*2,3 | 1,9*2,1 | | | | | |
| 12 | 3,4 | 1,2*2,5 | 1,4*2,4 | 1,6*2,3 | 1,10*2,1 | 2,6 | | | |
| 13 | 1,3*2,5 | 1,5*2,4 | 1,11*2,1 | | | | | | |
| 14 | 2,1*4,3 | 1,2*4,3 | 2,1*3,4 | 1,2*3,4 | 1,2*2,6 | 1,4*2,5 | 1,6*2,4 | 1,12*2,1 | 2,7 |
| 15 | 1,3*2,6 | 1,5*2,5 | 1,13*2,1 | | | | | | |
| 18 | 1,8*2,5 | 2,9 | | | | | | | |
| 20 | 2,10 | | | | | | | | |
| 24 | 2,12 | | | | | | | | |

To aid photographic reduction the type used in Table 2 does not differentiate between the running number of the polyhedron (bold face type in Table 5 of part I), the running number of the point group (italics in Table 3 of part I), and the symbol representing the *Z* terms. Thus 4*B*2*GM* **8–1** (*61,60,59*) appears in Table 2, without a possibility of confusion, as 4*B*2*GM* 8–1 (61,60,59).

### Reference

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